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A FINITE ELEMENT COMPOSITE BEAM MODEL FOR INTELLIGENT STRUCTURES APPLICATION

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Abstract. *This work explores the properties of some smart materials like piezoelectric materials or shape memory alloys for use in intelligent structures. The use of these materials has growing very fast in recent years, and today they occupy an important position in some high-technological applications, as spatial devices and biomechanics. The remarkable properties of these materials motivate their application in many situations, which may include among others, structural vibration control and control of deflection in structures. In this study it is presented a simple displacement-based finite element model of a composite beam with an embedded or bonded smart actuator and its behavior is studied considering the use of the actuator to control the beam actively or passively. The Bernoulli-Euler hypothesis is used, and the actuator can be used as a lamina or a wire. The possibilities explored are the use of the actuator for deflection control or vibration control, by changing its residual strain. An interesting aspect of the model is that the basic principles presented here can be easily extended to other structural elements, as plates, shells or Timoshenko beams.*

Key words: *Smart materials, Intelligent structures, Vibration control*

1. INTRODUCTION

The use of intelligent structures has increased very fast in the latest years. Although in present days there are a lot of structures and devices using these principles, the field of applications still offering many possibilities to be explored.

In recent years, some investigations have been concerned to research the aspects of composite structures with intelligent materials like Shape Memory Alloys and piezoelectrics bonded or embedded within. Rogers (Rogers *et al.*, 1991) and Brinson (Brinson *et al.*, 1996) have used models for plates and beams using SMA actuators respectively. Suleman and Venkaya (1995) and Park and Chopra (1996) have made similar models using piezoelectric actuators.

This work presents a very simple model for a bending beam with an actuator made of smart material (SMA or piezoelectric) using the Finite Element Model, and applying the Bernoulli-Euler hypothesis. The model can also be used in composites using elastic pre-strained alloys or materials.

The basis of this work is the ability of some materials to recover or develop residual strains when submitted to some external energy influence, as an electric field - case of piezoelectrics - or thermal loads - shape memory alloys. The shape memory alloys also have the ability to alter their elastic modulus when submitted to thermal variations (Scheckty and Wu, 1991).

More specific cases of deflection control and vibration control can be found in Saravanos and Heyliger (1995) and Savi (Savi *et al,* 1998). Drozdov and Kalamkarov (1996) made a general analytical study of residual strain and material property optimization.

This work is divided in 4 sections. In section 2 the model is presented and developed for general application. Section 3 is divided in two subsections, one referent to deflection control and other dealing with dynamical control. In section 4 the results are commented and some conclusions about the use of intelligent actuators in composite structures are presented.

2. MODELING

2.1 General considerations

This model assumes that the beam behaves as a Bernoulli-Euler beam in bending and can be made of various different laminae. The assumed displacement field for the structure is given by:

$$u_x = u + z\frac{\partial w}{\partial x} \qquad u_z = w(x) \tag{1}$$

where u is the longitudinal displacement of the midplane of the beam. Therefore, the longitudinal beam strain is given by:

$$\varepsilon = \frac{\partial u}{\partial x} + z\frac{\partial^2 w}{\partial x^2} = \varepsilon_0 + z\frac{\partial^2 w}{\partial x^2} \tag{2}$$

The stress σ_{xx} is related to the strain ε_{xx} by Hooke's law, σ_l are the stresses developed on the laminae and σ_a is the stress developed on the actuator. When the residual strain ε_a is activated, the stress developed on the actuator is given by Eq. (3b).

$$\sigma_l = E_l \varepsilon \qquad \text{stress on the laminae} \tag{3a}$$

$$\sigma_a = E_a(\varepsilon + \varepsilon_a) \qquad \text{stress on the actuator} \tag{3b}$$

The stress developed on the actuator can be divided into two different stresses: the elastic stress σ_e and the induced-strain stress σ_{is}, as shown in Eq. (4)

$$\sigma_a = \sigma_e + \sigma_{is} = E_a\varepsilon + E_a\varepsilon_a \tag{4}$$

In Eq. (4) E_a is the Young modulus of the actuator. The layers are distributed on the beam as shown in Fig. 1:

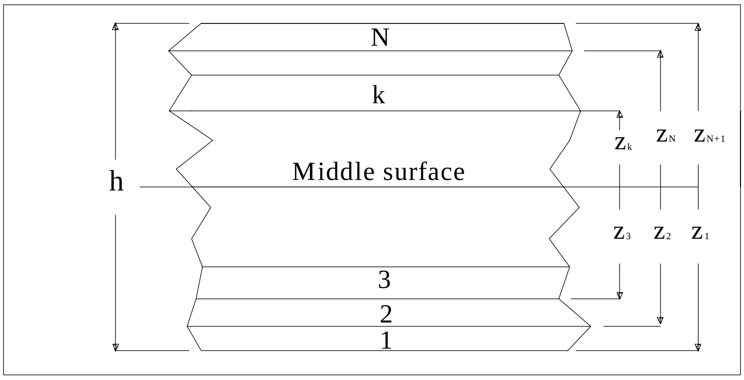


Figure 1 - Squematic view of the numbering system of the laminate

As it can be observed in Fig. 1, the beam can be constituted by N different laminae, which are numbered in sequence.

2.2 Composite beams

The longitudinal force N applied in the beam is expressed in terms of stresses as:

$$N = \int_A \sigma dA = \int_{A_m} \sigma_e dA + \int_{A_a} \sigma_{is} dA \tag{5}$$

In Eq. (5), A, A_m and A_a are the sectional areas of the beam, of the matrix and the actuator respectively. As the stress on the actuator is separated in elastic stress and induced strain stress, the elastic stress was included on the first integral of the right part of Eq. (5), and the second integral of the right part of Eq. (5) deals with the induced-strain stress. Equation (5) can be expanded to include stresses developed in each laminate of the beam, as shown in Eq. (6)

$$N = \int_A \sigma_e dA + \int_{A_a} \sigma_{si} dA = \sum_{k=1}^{N} \left(\int_{A_k} \sigma_k dA + \int_{A_a} \sigma_{si} dA \right) \tag{6}$$

In Eq. (6), the integral of stress in the area of the lamina was performed for each kth lamina, and when the laminate include or is an actuator, the second integral of Eq. (6) computes the longitudinal force developed by the actuator, when it is activated. If the laminate has no actuator, the induced-strain, and consequently the eventually induced stress is equal to zero. Performing the complete integration of Eq. (6), and considering Eqs. (1) through (4), one obtains the following equation for the longitudinal force in the beam (for simplicity, $\kappa_{xx} = \partial^2 w/\partial x^2$):

$$N = A_{11}\varepsilon_0 + B_{11}\kappa_{xx} + \varepsilon_a E_a A_a \tag{7}$$

The expression containing ε_a in Eq. (7) is obtained considering that the Young modulus of the actuators are all equal and the actuators develop the same induced-strain. A_a is the transverse area of the actuator(s) and A_{11} and B_{11} are the extensional stiffness and the coupling stiffness as defined in the Eqs. (8) :

$$A_{11} = \sum_{k=1}^{N} E_k b \int_{Z_{k-1}}^{Z_k} dz = \sum_{k=1}^{N} E_k A_k \tag{8a}$$

$$B_{11} = \sum_{k=1}^{N} E_k b \int_{Z_{k-1}}^{Z_k} z dz = \sum_{k=1}^{N} E_k H_k \tag{8b}$$

In Eqs. (8), b is the width of the beam, k is the number of the lamina, E_k is the Young modulus of the kth lamina, and A_k and H_k are the area and the static moment of the kth lamina respectively. The moment of a beam in function of the stresses developed in the beam is given by:

$$M = \int_A \sigma_e z dA + \int_{A_a} \sigma_{si} z dA = \sum_{k=1}^{N} \int_{A_k} \sigma_k z dA + \int_{A_a} \sigma_{si} z dA \tag{9}$$

Performing similar calculus as it was made in the case of longitudinal force, the following equation is obtained for the moment in the beam:

$$M = B_{11}\varepsilon_0 + D_{11}\kappa_{xx} + \varepsilon_a E_a H_a \tag{10}$$

In Eq. (10) H_a is the static moment of the actuator and D_{11} is the bending stiffness, as defined in Eq. (11).

$$D_{11} = \sum_{k=1}^{N} E_k b \int_{Z_{k-1}}^{Z_k} z^2 dz = \sum_{k=1}^{N} E_k I_k \tag{11}$$

In Eq. (11) I_k is the moment of inertia of the kth lamina. Based on equation (7), it is possible to write an expression for the longitudinal strain in the midplane ε_0:

$$\varepsilon_0 = \frac{\partial u}{\partial x} = \frac{N}{A_{11}} - \frac{B_{11}}{A_{11}} \frac{\partial^2 w}{\partial x^2} - \frac{\varepsilon_a E_a A_a}{A_{11}} \tag{12}$$

Substituting (12) in (10) one obtains:

$$M = \frac{B_{11}}{A_{11}}\left(N - B_{11}\frac{\partial^2 w}{\partial x^2} - \varepsilon_a E_a A_a \right) + D_{11}\frac{\partial^2 w}{\partial x^2} + \varepsilon_a E_a H_a \tag{13}$$

The governing equation of equilibrium for composite laminates beams using Bernoulli-Euler hypothesis is given by:

$$\frac{\partial^2 M}{\partial x^2} + N\frac{\partial^2 w}{\partial x^2} + q(x) = I_0 \frac{\partial^2 w}{\partial t^2} - I_2 \frac{\partial}{\partial t^2}\frac{\partial^2 w}{\partial x^2} + I_1 \frac{\partial^2}{\partial t^2}\frac{\partial u}{\partial x} \tag{14}$$

In Eq. (14) I_0, I_1 and I_2 are the rotary inertia terms and they are defined by the following general equation:

$$I_i = \sum_{k=1}^{N} \rho_k b \int_{Z_{k-1}}^{Z_k} z^i dz \qquad i = 0,1,2 \tag{15}$$

In Eq. (15) ρ_k is the mass density of the *k*th lamina. Thus, substituting Eqs. (12) and (13) in (14), and re-arranging some terms, a general equilibrium equation for the composite laminated beam is obtained:

$$\begin{aligned}&\left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2}{\partial x^2}\left(\frac{B_{11}}{A_{11}} N - \frac{B_{11}}{A_{11}}\varepsilon_a E_a A_a + \varepsilon_a E_a H_a\right) + N\frac{\partial^2 w}{\partial x^2} + q(x) = I_0 \frac{\partial^2 w}{\partial t^2} \\ &+ \left(I_1 \frac{B_{11}}{A_{11}} - I_2\right)\frac{\partial^2}{\partial t^2}\frac{\partial^2 w}{\partial x^2} + \frac{I_1}{A_{11}}\frac{\partial^2}{\partial t^2}\left(N - \varepsilon_a E_a A_a\right)\end{aligned} \tag{16}$$

It should be noted that if the beam is symmetric, all the terms involving H_a, I_1 and B_{11} vanishes.

2.3 Finite Element Model

Several approaches have been developed to use finite element in composite laminates, as can be found in Saravanos and Heyliger (1995), Baz and Chen (1995), Brinson and Lammering (1993) and Suleman and Venkaya (1995). Here it will be presented one of the simplest approaches, which is considerably efficient for the purpose of this work. Using a weight function $v(x)$ in Eq. (16) and integrating it between 0 and L, we obtain the following equation:

$$\int_L \begin{bmatrix} \left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)\frac{d^2 v}{dx^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2}{\partial x^2}\left(N\frac{B_{11}}{A_{11}} - \frac{B_{11}}{A_{11}}\varepsilon_a E_a A_a + \varepsilon_a E_a H_a\right) + N\frac{\partial v}{\partial x}\frac{\partial w}{\partial x} vq - \\ I_0 \frac{\partial v}{\partial x}\frac{\partial w}{\partial x} + \left(I_1 \frac{B_{11}}{A_{11}} - I_2\right)\frac{\partial v}{\partial x}\frac{\partial^3 w}{\partial x \partial t^2} \\ -\frac{I_1}{A_{11}} v \frac{\partial^2}{\partial t^2}\left(N - \varepsilon_a E_a A_a\right) \end{bmatrix} dx = 0 \tag{17}$$

Some boundary conditions relationed to the displacements are produced together Eq. (17), but they are dispensable for the scope of this work. Now, Considering $v(x) = \sum_{j=1}^{n} \varphi_j^e(x)$ and $w(x) = W^e(x) \equiv \sum_{j=1}^{n} w_j^e \cdot \varphi_j^e(x)$ and doing some algebraic operations, it is possible build the elementar system of the problem, as showed in Eq. (18):

$$\left(\left[K^e\right] + \left[G^e\right]\right)\left\{w^e\right\} + \left[M^e\right]\left\{\ddot{w}^e\right\} = \left\{F^e\right\} - \left\{F'^e\right\} \tag{18}$$

Where

$$
\begin{aligned}
K_{ij}^{e} &= \int_{L} \left(\frac{B_{11}^{2}}{A_{11}} - D_{11} \right) \frac{d^{2}\varphi_{i}}{dx^{2}} \frac{d^{2}\varphi_{i}}{dx^{2}} dx \\
M_{ij}^{e} &= \int_{L} \left[I_{0}\varphi_{i}\varphi_{j} + \left(I_{2} - I_{1}\frac{B_{11}}{A_{11}} \right) \frac{d\varphi_{i}}{dx}\frac{d\varphi_{j}}{dx} \right] dx \\
G_{ij}^{e} &= \int_{L} N \frac{d\varphi_{i}}{dx}\frac{d\varphi_{j}}{dx} dx \\
q_{i}^{e} &= \int_{L} \varphi_{i} q dx \\
F_{i}^{e} &= q_{i}^{e} + Q_{2i-1}\varphi_{i}(x_{i}) + Q_{2i+1}\varphi_{i}(x_{i+1}) \\
&\quad + Q_{2i}\left(-\frac{d\varphi_{i}}{dx} \right)_{x_{i}} + Q_{2i+2}\left(-\frac{d\varphi_{i}}{dx} \right)_{x_{i+1}}
\end{aligned}
\tag{19}
$$

$$
F_{i}^{\prime e} = \int_{L} \left(H_{a} - \frac{B_{11}}{A_{11}} A_{a} \right) \frac{d^{2}\varphi_{i}}{dx^{2}} E_{a}\varepsilon_{a} dx \tag{20}
$$

Equation (20) in vetorial form is given by:

$$
\{F^{\prime e}\} = \left(H_{a} - \frac{B_{11}}{A_{11}} A_{a} \right) E_{a}\varepsilon_{a} \begin{Bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{Bmatrix} \tag{21}
$$

Equation (21) is the elementar load vector representing the load developed in the beam when the residual strain ε_a is activated.

3. RESULTS

3.1 Deflection Control

To use the beam to control its deflection when submitted to static loads, the following global equation is used:

$$
[K]\{w\} = \{F\} - \{F'\} \tag{22}
$$

In Eq. (22) it can be seen that the difference between the usual global system for static loads and the model used in this work is the induced-strain load vector {F'}. When there is no induced-strain in the actuator, the load vector vanishes.

A squematic lateral view of the beam utilized to perform the results concerned to deflection control is showed in Fig 2. The beam has the following dimensions: 1000 mm length (L), 40 mm high (h) - 2mm of actuator (h_a) and 38 mm matrix - and 20 mm width (b).

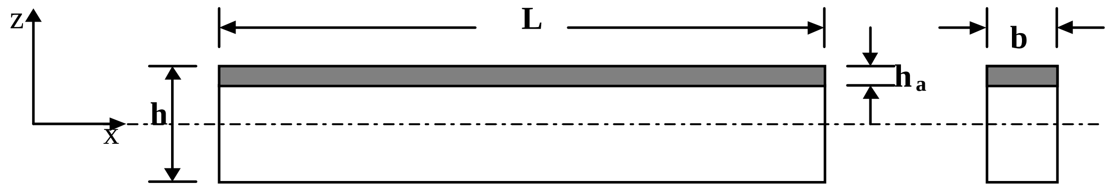


Figure 2- Squematic lateral view of the beam utilized on the numerical modeling

Figure 3 shows the three kinds of external loads utilized to perform the results and the relative position of the actuator on the beam.

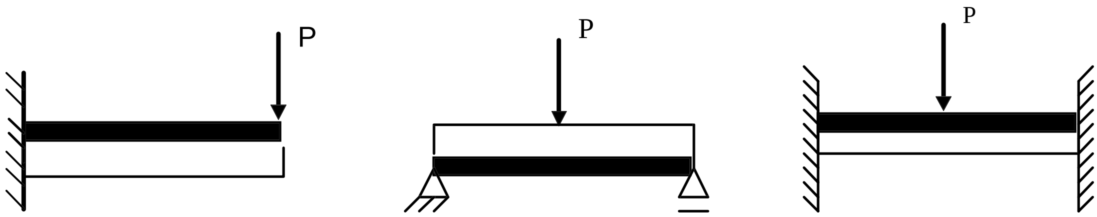


Figure 3 - (a) cantilevered beam; (b) simply supported; and (c) clamped-clamped beam

The material properties of the beam (actuator and matrix) are showed in Table 1, and in Table 2 it is showed the maximum (in module) transverse deflections of the beam with inactivated actuator comparing with activated-actuator deflections.

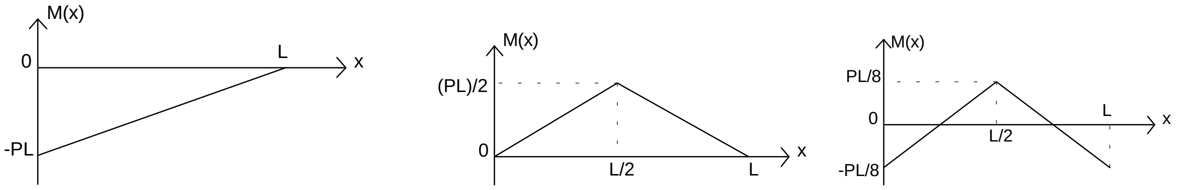
Table 1. Elastic modulus and mass density of the actuator and the matrix

	actuator	matrix
Elastic Modulus (GPa)	65,12	3,00
Mass density (Kg.10^3/m^3)	5,00	1,00

Table 2. Maximums deflections of the beam with the actuator activated and inactivated

ε_a = -0,002976	cantilevered beam		simply supported beam		Clamped Beam	
actuator	*inact.*	*Activated*	*inact.*	*activated*	*inact.*	*activated*
Number of Elements	1	1	2	2	2	2
Numerical Results (mm)	-87,53	40,02	-5,47	6,40	-1,37	-1,37
Analytical Results (mm)	-87,53	40,02	-5,47	6,40	-1,37	-1,37

From Table 2, it can be observed that the cantilevered beam and the simply supported beam changed their transverse deflections, which became positives. In the cantilevered beam, the actuator was located on the upper surface of the beam (as shown in Fig. 1), and in the simply supported beam, the actuator was located in the lower surface. It was made because the induced stresses developed in the actuator were negative, which produced compressive induced stresses. In fact, it can be observed that the sign of the moment can be associated with the type of stress or strain developed on each surface of an elastic beam. Figure 4 shows the bending diagrams of the beam:



(a) (b) (c)

Figure 4 - Bending Diagrams of (a) Cantilevered beam; (b) simply supported beam and (c) clamped beam

As established here, the negative sign of the bending moment means that the stress on the superior surface is tensile stress, and a positive sign design compressive stress. In view of this, it is very important locate the actuator in the surface according the kind of stress developed on that surface. If the residual strain of the actuator is obtained through tensile stress, the actuator must be putted on the surface with tensile stress.

In the case of a cantilevered beam, the upper layer of the beam is submitted to a tensile stress, therefore the actuator with tensile strain must be placed in its upper layer. We can see that the clamped-clamped beam has diagram with both positive and negative signs, and if the moment is integrated across the length of the beam, it is possible to see that this integration is equal to zero. Therefore the transverse deflection of the beam did not changed in activating the actuator.

Following the ideas descripted above, its possible to predict where is the better place to locate the actuator analyzing the sign of the curvature, or the bending moment diagram.

3.2 Dynamical Control

The induced-strain can be also used for vibration control. As it can be seem in Eq. (14), the normal force across the beam acts on the dynamic response of the beam. Therefore, the induced-strain developed on the actuator can be used to generate a normal force across the beam, controlling the dynamical response of the beam.

In view of this, a basic study can be made exploring the free response of the beam, i.e., the control of natural frequencies of the beam. The elementar equation to study these properties is given by Eq. (23).

$$\left(\left[K^e\right]+\left[G^e\right]-\omega^2\left[M^e\right]\right)\left\{W^e\right\}=\{0\} \tag{23}$$

where W^e are the nodal values of the dynamical displacement (Reddy, 1997), and the displacement in the *z* axis (*w*) is considered periodic in time, i.e., $\omega_j^e(t)=W_j^e e^{-i\omega t}$. The normal force N is given by Eq. (7), and in the absence of external loads, it can be written as

$$N=\varepsilon_a E_a A_a$$

To perform the dynamical control of the beam, the following simply supported beam (Fig. 5) configuration was considered:

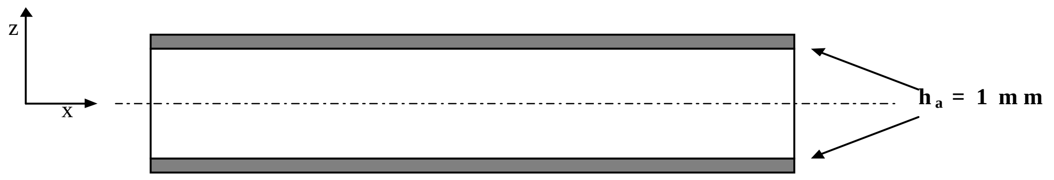


Figure 5 - Configuration used to perform dynamical results

In Table 3 it is showed the results obtained through the use of the actuator as a generator of normal force across the beam. The induced-strain used is of 0.3%, positive and negative, which is a reasonable value for many materials. The numerical results are compared with the theoretical ones, to illustrate the effectiveness of the model.

Table 3. Natural (fundamental) frequencies of the beam

Natural Frequency (Hz)			
N. of elements	Inactivated ($\varepsilon_a = 0$)	Activated ($\varepsilon_a = 0.003$)	Activated ($\varepsilon_a = -0.003$)
2	359.381	457.586	221.187
4	358.061	456.518	219.105
10	357.971	456.447	218.958
Analytical	357.968	456.445	218.954

In Table 3 the results obtained using positive strains - and positive or tensile longitudinal force N - show an increase of the natural frequencies. These results are in agreement with the ones obtained in the current literature (Reddy, 1997). By other side, when N is negative or compressive, the natural frequencies decrease.

Another possible kind of dynamical control using smart actuators is the alteration of their mechanical properties, like the Young modulus. This is specifically true for Shape Memory Alloys, which have the ability to change their Elastic modulus depending on their crystalline structure (Savi *et al*, 1998). Some nickel-titanium alloys can increase their elastic modulus about three to four times greater when transforms from austenitic to martensitic phase.

4. CONCLUSIONS

This work presented and performed a finite element model of a beam using an actuator with a residual strain ε_a that can be activated to generate axial stresses in the actuator and consequently act on the beam to control its deflection or vibration. The obtained results presented a very satisfactory agreement with respect to the analytical results.

Some considerations can be deduced from this work, which the most important are: the finite element formulation is quite simple, and for example, to deflection control it consists basically in a insertion of a load vector, very similarly as it is usually made when thermal loads are considered (Reddy, 1997); the position of the actuator in the beam is very important, and must be analyzed before its application, considering the kind of stresses that will be developed on the beam; the model is easily applicable in dynamical control, exploring the induced-strain to generate an axial load and control the dynamical response of the beam.

Finally, all the aspects commented above can be extended to more complex structural elements, as Timoshenko beams, plates and shells.

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